# worksheet 8 solutions

Wednesday, March 10, 2021 8:54 AM

**problem 1**

$f(x) = \frac{2}{x-4}\, a = 0, b = .5$

Trap:

$\frac{(b-a)}{2}\big(f(a)+f(b)\big) = \frac{1}{4}\left(-\frac{1}{2}+\frac{-2}{3.5}\right) = -0.2678571428571428$

$h = (b-a)$

error:

$\frac{h^3}{12} f''(\xi)$

$f'' = \frac{4}{(-4+x)^3}$

maximized at right endpoint

$|f''| \leq 0.09329446064139942$

$\left|\frac{h^3}{12} f''(\xi)\right| \leq 0.0009718172983479105$

actual error: 0.0007943576081004822

Simpson:

$h = \frac{b-a}{2}$

$\frac{h}{3}\big(f(x_0)+4f(x_1)+f(x_2)\big) = -0.26706349206349206$

Error: $\frac{h^5}{90} f^{(4)}(\xi)$

$f^{(4)}(x) = \frac{48}{(-4+x)^5}$

maximized on right endpoint:

$\left|f^{(4)}(x)\right| \leq \left|f^{(4)}(.5)\right| = 0.09139049205688106$

therefore:

$\left|\frac{h^5}{90} f^{(4)}(\xi)\right| \leq 9.916503044366435 \times 10^{-7}$

actual error: $7.068144496913398 \times 10^{-7}$

midpoint:

$\int_a^b f(x)dx = 2h\, f\left(\frac{b-a}{2}\right) + \frac{h^3}{3} f''(\xi)$

$$h = \frac{b-a}{2}$$
get: $-0.26666666666666666$

error is bounded by 0.00048590864917395527
actual error is: 0.0003961185823757041

**Problem 2**

$$4 = f(0) + f(2)$$
$$2 = \frac{1}{3}(f(0) + 4f(1) + f(2))$$

so
$$6 = f(0) + 4f(1) + f(2)$$

$$2 = 4f(1)$$
$$f(1) = \frac{1}{2}$$

**Problem 3**

We require:
$$1 = \int_0^1 1\,dx = c_0 + c_1$$
$$\frac{1}{2} = \int_0^1 x dx = c_1 x_1$$
$$\frac{1}{3} = \int_0^1 x^2 dx = c_1 x_1^2$$

(this is the most we can hope for, but we can always see if our scheme works for $x^4$)

note $x_1 \neq 0$, so we can divde the third equation by the second:

$$\frac{2}{3} = x_1$$
then we can solve the second equation
$$c_1 = \frac{3}{4}$$
therefore
$$c_0 = \frac{1}{4}$$

now $\frac{1}{4} = \int_0^1 x^3\,dx \neq c_1 x_1^3 = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)^3 = \frac{24}{108}$
so we found the best possible thing.

**Problem 4**

$$f(x) = e^{2x}\sin(3x)$$
$$a = 0$$
$$b = 2$$